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NUMERICAL STUDY OF 3D PULSATING SOURCE GREEN FUNCTION OF FINITE WATER DEPTH

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ABSTRACT
The accurate and efficient evaluation of the Green function and its derivatives for a pulsating source in finite water depth is one of the most important aspects in wave force calculation for offshore structures, at the same time it is also one of the most challenging tasks due to the singularity in the Cauchy principal value integral and the oscillation behavior of the Bessel function. In this paper, a new integral equation is proposed in which the singular term is deducted from the Green function. Furthermore, the Gauss-Laguerre integral equation proposed by other researcher has been improved to obtain a new form of the equation. Using these two proposed methods, numerical calculations are performed for the pulsating source Green function and its derivatives for finite water depth. The results show that very good agreements are achieved between the present results and other published data. The precision and efficiency of the present methods are also investigated and compared with the series solution and traditional Gauss-Laguerre integral method. It shows that both of the new methods have better precision than the traditional Gauss-Laguerre integral, but less efficient than the series solution. On the other side, the series solution would lose precision in the near-fields approaching zero, but the new Gauss-Laguerre integral equation could obtain right results. Furthermore the series solution has poor precision in large wave frequency and water depth in which case both of the new methods could obtain right results. Finally, one strategy has been proposed which could properly obtain the value of green function and its derivatives.

1 INTRODUCTION
The water depth is always shallow near shore and island, and the hydrodynamic motion and load response in shallow water have big difference compared with deep water depth. So it needs to study the wave motion and load response of floating body in shallow water. During computing the motion of floating body, the biggest difficulty lies in accurately solving Green function in finite water depth. Only the precise solution of green function and its partial derivatives in finite water depth are obtained, it's possible to get the right motion response of floating structures. The expression of green function in finite water depth has two forms. One is integral form [1]; the other is series solution [2]. The former one has high computing precision; meanwhile is applicable in both near-field and far-field, but has low efficiency. The later one has high efficiency, but is difficult to converge in the vicinity of the near-field, especially the existence of a singularity at $R = 0$. Therefore, for calculating the finite water depth Green function and its derivatives the researchers generally use the integral form in near-field, use the series solution form in far-field. In 1984 Newman [3] gave deep discussions of solving the green function in finite water depth and infinite water depth, including integral solution and the series solution of finite water depth, but it did not give a specific numerical method for solving integral form. Li [4], Xie [5] and Liu [6] have given some

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numerical methods to calculate the value of green function and its derivatives, furthernore Xie [5] and Liu [6] have shown some results of hydrodynamic motion or loads. The numerical methods of the integral form generally are divided into two types, one is curve fitting; the other is numerical integral directly. For example, the Hydrostar from BV is using Chebyshev polynomial fitting. Direct numerical integral is traditionally using Gauss-Laguerre integral (Li [4] and Liu[6]). Although Li [4], Xie [5] and Liu [6] have proposed some numerical integral methods, but the accuracy and speed of these methods need to be verified. The paper here has improved the calculating precision of the method initially proposed by Liu [6]. At the same time a new method is initially proposed here after a certain derivation. Finally the investigation on the accuracy among the two methods proposed in the paper and series solution has been carried out. Furthermore the calculating velocity also has been studied. The results and conclusions have a certain reference meaning in solving the green function and its derivatives in finite water depth.

2 NUMERICAL SOLVING METHOD OF RADIATION POTENTIAL

The boundary conditions of solving radiation potential in uniform finite water depth is

[L] $\nabla^2\phi_j(x,y,z)=0$ In fluid,

[F] $\dfrac{\partial\phi_j}{\partial z}-\nu\phi_j=0$ On the free surface,

[S] $\left.\dfrac{\partial\phi_j}{\partial n}\right|_{S_0}=n_j$ On the wet body,

[B] $\left.\dfrac{\partial\phi_j}{\partial z}\right|_{z=-H}=0$ In the bottom,

[R] $\lim\limits_{R\to\infty}\sqrt{R}\left(\dfrac{\partial\phi_j}{\partial R}-ik\phi_j\right)=0$ Radiation condition

For satisfying the boundary condition of free surface, radiation condition in far-field and water bottom impenetrable conditions, the expression of Green function is adopted as follows

Integral form [1]

$$G(x,y,z,\xi,\eta,\zeta)=\frac{1}{r}+\frac{1}{r_2}+$$

$$\text{P.V.}\int_0^\infty\frac{2(k+\nu)\mathrm{e}^{-kH}\cosh k(z+H)\cosh k(\zeta+H)}{k\sinh(kH)-\nu\cosh(kH)}\mathrm{J}_0(kR)\mathrm{d}k+$$

$$\mathrm{i}\frac{2\pi(K+\nu)\mathrm{e}^{-KH}\sinh(KH)\cosh K(z+H)\cosh K(\zeta+H)}{\nu H+\sinh^2(KH)}\mathrm{J}_0(KR)\quad(1)$$

Series solution [2]

$$G(x,y,z,\xi,\eta,\zeta)=2\pi\mathrm{i}\frac{K^2-\nu^2}{H(K^2-\nu^2)+\nu}\cosh K(z+H)$$

$$\cdot\cosh K(\zeta+H)\cdot\mathrm{H}_0^{(1)}(KR)+4\sum_{n=1}^{\infty}\frac{k_n^2+\nu^2}{H(k_n^2+\nu^2)-\nu}$$

$$\cdot\cos k_n(z+H)\cdot\cos k_n(\zeta+H)\mathrm{K}_0(k_nR)\quad(2)$$

where, H, K and ω respectively are water depth, wave number and frequency. $K\tanh(KH)=\nu$. $\nu=\dfrac{\omega^2}{g}$. (ξ,η,ζ) and (x,y,z) are source point and field point. $R=\sqrt{(x-\xi)^2+(y-\eta)^2}$. $r=\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}$. $r_2=\sqrt{(x-\xi)^2+(y-\eta)^2+(z+2H+\zeta)^2}$. k_n is the plus root of the equation $k_n\tan k_nH+\nu=0$. $\mathrm{H}_0^{(1)}(kR)$ is the first kind Hankel function with order zero. $\mathrm{K}_0(k_nR)$ is modified Bessel function of second kind with order zero. $\mathrm{J}_0(kR)$ is first kind Bessel function with zero order. $\dfrac{1}{r_2}$ is the mirror of $\dfrac{1}{r}$ about bottom.

For convenience, the integral part of the principal value of Green function and its derivatives are as follows

$$G_0=\text{P.V.}\int_0^\infty\frac{2(k+\nu)\mathrm{e}^{-kH}\cosh k(z+H)\cosh k(\zeta+H)}{k\sinh(kH)-\nu\cosh(kH)}\mathrm{J}_0(kR)\mathrm{d}k\quad(3)$$

$$G_R=\frac{\partial G_0}{\partial R}=-\text{P.V.}\int_0^\infty\frac{2k(k+\nu)\mathrm{e}^{-kH}\cosh k(z+H)\cosh k(\zeta+H)}{k\sinh(kH)-\nu\cosh(kH)}\mathrm{J}_1(kR)\mathrm{d}k\quad(4)$$

$$G_z=\frac{\partial G_0}{\partial z}=\text{P.V.}\int_0^\infty\frac{2k(k+\nu)\mathrm{e}^{-kH}\sinh k(z+H)\cosh k(\zeta+H)}{k\sinh(kH)-\nu\cosh(kH)}\mathrm{J}_0(kR)\mathrm{d}k\quad(5)$$

The time-consuming of calculating the value of Green function and its derivatives has occupied the most time of the whole procedure of solving velocity potential by using BEM in equations (3)-(5) [7]. Thus, it's necessary to study the numerical methods to solve the values of these equations. The procedure is shown as follows.

2.1 Subsection Integral

2.1.1 Value of Green Function

For non-dimension, it introduces $\bar{k}=kH$, $\bar{\nu}=\nu H$, $\bar{R}=R/H$, $\bar{z}=z/H$, $\bar{\zeta}=\zeta/H$, then

$$\overline{G_0}=G_0\cdot H=\text{P.V.}\int_0^\infty\frac{2(\bar{k}+\bar{\nu})\mathrm{e}^{-\bar{k}}\cosh\bar{k}(\bar{z}+1)\cosh\bar{k}(\bar{\zeta}+1)}{\bar{k}\sinh\bar{k}-\bar{\nu}\cosh\bar{k}}\mathrm{J}_0(\bar{k}\cdot\bar{R})\mathrm{d}\bar{k}\quad(6)$$

For convenience, it introduces

$$f(\bar{k})\equiv2(\bar{k}+\bar{\nu})\mathrm{e}^{-\bar{k}}\cosh\bar{k}(\bar{z}+1)\cosh\bar{k}(\bar{\zeta}+1)\mathrm{J}_0(\bar{k}\cdot\bar{R})\quad(7)$$

$$p(\bar{k})\equiv\bar{k}\sinh\bar{k}-\bar{\nu}\cosh\bar{k}\quad(8)$$

$$g(\bar{k})\equiv\frac{f(\bar{k})(\bar{k}-\bar{k}_0)}{p(\bar{k})}\quad(9)$$

where $\bar{k}_0$ is the plus root of equation $\bar{k}\tanh\bar{k}-\bar{\nu}=0$, so $\bar{k}_0=K\cdot H$.

Then $\overline{G_0}$ becomes into

$$\overline{G_0} = \text{P.V.}\int_0^{\infty} \frac{g(\bar{k})}{\bar{k}-\bar{k}_0}\mathrm{d}\bar{k} = \int_0^{2\bar{k}_0} \frac{g(\bar{k})-g(\bar{k}_0)}{\bar{k}-\bar{k}_0}\mathrm{d}\bar{k} + g(\bar{k}_0)\left[\text{P.V.}\int_0^{2\bar{k}_0} \frac{1}{\bar{k}-\bar{k}_0}\mathrm{d}\bar{k}\right] + \int_{2\bar{k}_0}^{\infty} \frac{g(\bar{k})}{\bar{k}-\bar{k}_0}\mathrm{d}\bar{k} \tag{10}$$

For $p(\bar{k}_0) \equiv \bar{k}_0 \sinh \bar{k}_0 - \bar{\nu}\cosh \bar{k}_0 = 0$, so $p(\bar{k})$ could be expanded at $\bar{k}_0$ as follows

$$p(\bar{k}) \approx p(\bar{k}_0) + (\bar{k}-\bar{k}_0)\, p'(\bar{k}) = (\bar{k}-\bar{k}_0) \cdot \left[\sinh \bar{k} + \bar{k}\cosh \bar{k} - \bar{\nu}\sinh \bar{k}\right] \tag{11}$$

So

$$g(\bar{k}_0) \equiv \frac{f(\bar{k}_0)}{\sinh \bar{k}_0 + \bar{k}_0 \cosh \bar{k}_0 - \bar{\nu}\sinh(\bar{k}_0)} = \frac{\bar{k}_0 f(\bar{k}_0)/\cosh \bar{k}_0}{\bar{k}_0^2 - \bar{\nu}^2 + \bar{\nu}} \tag{12}$$

Because

$$\text{P.V.}\int_0^{2\bar{k}_0} \frac{1}{\bar{k}-\bar{k}_0}\mathrm{d}\bar{k} = 0 \tag{13}$$

Finally, $\overline{G_0}$ becomes

$$\overline{G_0} = \text{P.V.}\int_0^{\infty} \frac{g(\bar{k})}{\bar{k}-\bar{k}_0}\mathrm{d}\bar{k} = \int_0^{2\bar{k}_0} \frac{g(\bar{k})-g(\bar{k}_0)}{\bar{k}-\bar{k}_0}\mathrm{d}\bar{k} + \int_{2\bar{k}_0}^{\infty} \frac{f(\bar{k})}{p(\bar{k})}\mathrm{d}\bar{k} \tag{14}$$

The values of the both terms on the right of the above equation could be obtained by many numerical integral methods, such as Simpson, Romberg and Gauss integral. For the upper limit of the second term is infinite, it needs to truncate the infinite integral zone into finite integral zone. Here, two methods have been proposed, one is interval truncation method; the other is interval transformation.

(1) Interval truncation method

Introducing the following expression：

$$\int_{2\bar{k}_0}^{\infty} \frac{f(\bar{k})}{p(\bar{k})}\mathrm{d}\bar{k} = \sum_{n=1}^{\infty} I_n \tag{15}$$

where

$$I_n = \int_{k_n}^{k_{n+1}} \frac{f(\bar{k})}{p(\bar{k})}\mathrm{d}\bar{k}, \tag{16}$$

$$k_1 = 2\bar{k}_0,\quad k_{n+1} = k_n + \delta,\ n = 1, 2, 3, \ldots$$

Generally $\delta = 1.0$. The sum of some parts is $S_M = \sum_{n=1}^{M} I_n$. When $|I_{M+1}/S_M| \le \text{eps}$, the iteration converges and stops. If the compute performance is enough high, it could set $\text{eps} = 10^{-6}$. Additionally, for improving the calculating velocity, it's better set $\delta = n$.

(2) Interval transformation

Introducing variable $t = 1/\bar{k}$, then

$$\int_{2\bar{k}_0}^{\infty} \frac{f(\bar{k})}{p(\bar{k})}\mathrm{d}\bar{k} = \int_0^{1/(2\bar{k}_0)} \frac{f(1/t)}{t^2 \cdot p(1/t)}\mathrm{d}t = \int_0^{1/(2\bar{k}_0)} \frac{2(1+\bar{\nu}t)\mathrm{e}^{-1/t}\cosh\left(\frac{\bar{z}+1}{t}\right)\cosh\left(\frac{\bar{\zeta}+1}{t}\right)\mathrm{J}_0(\bar{R}/t)}{\sinh(1/t) - \bar{\nu}t\cosh(1/t)}\mathrm{d}t \tag{17}$$

Now it could obtain the value of the above expression by Gauss integral method.

2.1.2 The Derivatives of Green Function

Introducing non-dimensional variables $\bar{x} = x/H$, $\bar{\xi} = \xi/H$, $\bar{y} = y/H$, $\bar{\eta} = \eta/H$, $\bar{z} = z/H$, $\bar{\zeta} = \zeta/H$.

(1) $\frac{\partial G_0}{\partial R}$, $\frac{\partial G_0}{\partial x}$ and $\frac{\partial G_0}{\partial y}$

There is non-dimensional variable

$$\overline{G_R} = G_R \cdot H^2 = -\text{P.V.}\int_0^{\infty} \frac{2\bar{k}(\bar{k}+\bar{\nu})\mathrm{e}^{-\bar{k}}\cosh \bar{k}(\bar{z}+1)}{\bar{k}\sinh \bar{k} - \bar{\nu}\cosh \bar{k}} \cdot \cosh \bar{k}(\bar{\zeta}+1)\mathrm{J}_1(\bar{k}\cdot\bar{R})\mathrm{d}\bar{k} \tag{18}$$

When $R \neq 0$,

$$\overline{G_x} = G_x \cdot H^2 = \overline{G_R} \cdot \frac{\mathrm{d}\bar{R}}{\mathrm{d}\bar{x}} = -\frac{(\bar{x}-\bar{\xi})}{\bar{R}} \cdot \text{P.V} \cdot \int_0^{\infty} \frac{2\bar{k}(\bar{k}+\bar{\nu})e^{-\bar{k}}\cosh \bar{k}(\bar{z}+1)\cosh \bar{k}(\bar{\zeta}+1)}{\bar{k}\sinh \bar{k} - \bar{\nu}\cosh \bar{k}}\mathrm{J}_1(\bar{k}\cdot\bar{R})\mathrm{d}\bar{k} \tag{19}$$

$$\overline{G_y} = G_y \cdot H^2 = \overline{G_R} \cdot \frac{\mathrm{d}\bar{R}}{\mathrm{d}\bar{y}} = -\frac{(\bar{y}-\bar{\eta})}{\bar{R}} \cdot \text{P.V} \cdot \int_0^{\infty} \frac{2\bar{k}(\bar{k}+\bar{\nu})e^{-\bar{k}}\cosh \bar{k}(\bar{z}+1)\cosh \bar{k}(\bar{\zeta}+1)}{\bar{k}\sinh \bar{k} - \bar{\nu}\cosh \bar{k}}\mathrm{J}_1(\bar{k}\cdot\bar{R})\mathrm{d}\bar{k} \tag{20}$$

When R = 0, for $\frac{\mathrm{J}_1(\bar{k}\cdot\bar{R})}{\bar{k}\cdot\bar{R}} = \frac{1}{2}$, there are

$$\overline{G_x} = G_x \cdot H^2 = -(\bar{x}-\bar{\xi}) \cdot \text{P.V} \cdot \int_0^{\infty} \frac{\bar{k}^2(\bar{k}+\bar{\nu})\mathrm{e}^{-\bar{k}}\cosh \bar{k}(\bar{z}+1)\cosh \bar{k}(\bar{\zeta}+1)}{\bar{k}\sinh \bar{k} - \bar{\nu}\cosh \bar{k}}\mathrm{d}\bar{k} \tag{21}$$

$$\overline{G_y} = G_y \cdot H^2 = -(\bar{y}-\bar{\eta}) \cdot \text{P.V} \cdot \int_0^{\infty} \frac{\bar{k}^2(\bar{k}+\bar{\nu})\mathrm{e}^{-\bar{k}}\cosh \bar{k}(\bar{z}+1)\cosh \bar{k}(\bar{\zeta}+1)}{\bar{k}\sinh \bar{k} - \bar{\nu}\cosh \bar{k}}\mathrm{d}\bar{k} \tag{22}$$

(2) $\frac{\partial G_0}{\partial z}$

There is non-dimensional variable

$$\overline{G_z} = G_z \cdot H^2 = \text{P.V.}\int_0^{\infty} \frac{2\bar{k}(\bar{k}+\bar{\nu})\mathrm{e}^{-\bar{k}}\sinh \bar{k}(\bar{z}+1)\cosh \bar{k}(\bar{\zeta}+1)}{\bar{k}\sinh \bar{k} - \bar{\nu}\cosh \bar{k}} \cdot \mathrm{J}_0(\bar{k}\cdot\bar{R})\mathrm{d}\bar{k} \tag{23}$$

2.2 Gauss-Laguerre Integral method

2.2.1 Traditional Integral method

The traditional integral method is to isolate singularity during calculating the value of Green function and its derivatives and using Gauss-Laguerre integral method. The detailed procedure is as follows, here only shows the value of G_0, the approaches of obtaining G_R and G_z have the similar way.

Taken $P(k)=k\sinh(kH)-\nu\cosh(kH)$, where $P(K)=0$, then its derivative is

$$P'(k)=\sinh(kH)+kH\cosh(kH)-\nu H\sinh(kH) \quad (24)$$

For convenience, it introduces

$$Q(k)=2(k+\nu)\cosh k(z+H)\cosh k(\zeta+H)\mathrm{J}_0(kR) \quad (25)$$

Combining equations （3）and（24）, it could obtain the final form of the equation (3).

$$G_0=\int_0^\infty \mathrm{e}^{-kH}\left[\frac{Q(k)}{P(k)}-\frac{Q(K)}{(k-K)P'(K)}\right]\mathrm{d}k+\frac{Q(K)}{P'(K)}\mathrm{P.V.}\int_0^\infty \frac{\mathrm{e}^{-kH}}{k-K}\mathrm{d}k \quad (26)$$

The first term of the above expression could be calculated directly by Gauss-Laguerre integral method. The second term is exponential integral. Then

$$G_0=\sum_{i=1}^{n}\frac{w_i}{H}\left[\frac{Q(k_i)}{P(k_i)}-\frac{Q(K)}{(k_i-K)P'(K)}\right]-\frac{Q(K)}{P'(K)}\mathrm{e}^{-KH}\mathrm{E}_i(KH) \quad (27)$$

where w_i, x_i and n respectively are the weight, position and term number of Gauss-Laguerre integral. $k_i=x_i/H$. $\mathrm{E_i}()$ is function of exponential integral. The above expression is called traditional Gauss-Laguerre integral in finite water depth. For the oscillation of Bessel function, the convergence rate of the above expression is very slow. Generally, it needs 64 Gauss points to obtain enough precision; meanwhile, this method is very time-consuming and when $k_i << K$, it lets $\frac{Q(K)}{(k_i-K)P'(K)}$ is much larger than $Q(k_i)/P(k_i)$ in high frequency. The main reason is $\mathrm{e}^{K(z+\zeta+2H)}$ much larger than $\mathrm{e}^{k_i(z+\zeta+2H)}$. Thus it leads to small value term minus large value term, then plus large value term. But the small value term is desired term, so it causes losing precision in high frequency.

On the other side, the relative expressions only need change as follows during processing G_R and G_z.

$$Q_R(k)=-2k(k+\nu)\cosh k(z+H)\cosh k(\zeta+H)\mathrm{J}_1(kR) \quad (28)$$

$$Q_z(k)=2k(k+\nu)\sinh k(z+H)\cosh k(\zeta+H)\mathrm{J}_0(kR) \quad (29)$$

2.2.2 Improved Integral Method

Although Liu [6] has given one method to improve the precision based on traditional Gauss-Laguerre integral method, the method has some problem on processing the term of $\mathrm{e}^{k(z+\zeta+H)}$, in which the other terms containing exponential part are not handled that would cause error in the same way. Additionally, the derivatives of Green function are not need to isolate the term of 2ν. So a new improved method is proposed by the paper based on Liu [6] which is shown as follows.

For the error in traditional Gauss-Laguerre integral method is caused by exponential term, it could isolate exponential term to solve this problem.

$$G_0=\mathrm{P.V.}\int_0^\infty \frac{2(k+\nu)\mathrm{e}^{-kH}\cosh k(z+H)\cosh k(\zeta+H)}{k\sinh(kH)-\nu\cosh(kH)}\mathrm{J}_0(kR)\mathrm{d}k$$

$$=\mathrm{P.V.}\int_0^\infty \mathrm{e}^{-kH}\left[1+\frac{2\nu+(k+\nu)\mathrm{e}^{-2kH}}{[k\tanh(kH)-\nu](1+\mathrm{e}^{-2kH})}\right]\cdot\left[\mathrm{e}^{k(z+\zeta+H)}+\mathrm{e}^{-k(-z+\zeta+H)}+\mathrm{e}^{-k(z-\zeta+H)}+\mathrm{e}^{-k(z+\zeta+3H)}\right]\mathrm{J}_0(kR)\mathrm{d}k \quad (30)$$

$$G_R=-\mathrm{P.V.}\int_0^\infty \mathrm{e}^{-kH}\left[k+k\frac{2\nu+(k+\nu)\mathrm{e}^{-2kH}}{[k\tanh(kH)-\nu](1+\mathrm{e}^{-2kH})}\right]\cdot\left[e^{k(z+\zeta+H)}+\mathrm{e}^{-k(-z+\zeta+H)}+\mathrm{e}^{-k(z-\zeta+H)}+\mathrm{e}^{-k(z+\zeta+3H)}\right]\mathrm{J}_1(kR)\mathrm{d}k \quad (31)$$

$$G_z=\mathrm{P.V.}\int_0^\infty \mathrm{e}^{-kH}\left[k+k\frac{2\nu+(k+\nu)\mathrm{e}^{-2kH}}{[k\tanh(kH)-\nu](1+\mathrm{e}^{-2kH})}\right]\cdot\left[\mathrm{e}^{k(z+\zeta+H)}+\mathrm{e}^{-k(-z+\zeta+H)}-\mathrm{e}^{-k(z-\zeta+H)}-\mathrm{e}^{-k(z+\zeta+3H)}\right]\mathrm{J}_0(kR)\mathrm{d}k \quad (32)$$

The terms without singularity in equations (30) and (31) could be obtained by the following expressions.

$$\int_0^\infty \mathrm{e}^{-ax}\mathrm{J}_0(bx)\mathrm{d}x=\frac{1}{\sqrt{a^2+b^2}},\ \int_0^\infty x\mathrm{e}^{-ax}\mathrm{J}_0(bx)\mathrm{d}x=\frac{a}{(a^2+b^2)^{3/2}}$$

$$\int_0^\infty \mathrm{e}^{-ax}\mathrm{J}_1(bx)\mathrm{d}x=\frac{1}{b}\left(1-\frac{a}{\sqrt{a^2+b^2}}\right),\ \int_0^\infty x\mathrm{e}^{-ax}\mathrm{J}_1(bx)\mathrm{d}x=\frac{b}{(a^2+b^2)^{3/2}} \quad (33)$$

For the equations from (30) to (32) contain the exponential terms of $\mathrm{e}^{k(z+\zeta+H)}$, $\mathrm{e}^{-k(-z+\zeta+H)}$, $\mathrm{e}^{-k(z-\zeta+H)}$ and $\mathrm{e}^{-k(z+\zeta+3H)}$, the direct Gauss-Lauerre integral would have the same problems with the method of traditional Gauss-Lauerre integral. Thus, it needs some special processes. Liu [6] just recognized that it only needs to deal with the part containing $e^{k(z+\zeta+H)}$, but when k_i has big difference with K in high frequency, the value of each part containing exponential would have very big difference which could lead to lose precision in calculating the value of Green function.

The following procedure has taken $\mathrm{e}^{k(z+\zeta+H)}$ of G_0 as example, the others containing exponential terms have the similar approaches. The expression is

$$G_{01}=\mathrm{P.V.}\int_0^\infty \mathrm{e}^{-kH}\frac{2\nu+(k+\nu)\mathrm{e}^{-2kH}}{[k\tanh(kH)-\nu](1+\mathrm{e}^{-2kH})}\mathrm{e}^{k(z+\zeta+H)}\mathrm{J}_0(kR)\mathrm{d}k \quad (34)$$

Noting $P(k)=[k\tanh(kH)-\nu](1+\mathrm{e}^{-2kH})$, where $P(K)=0$, then its derivative is

$$P'(k)=\left[\tanh(kH)+kH\left[1-\tanh^2(kH)\right]\right](1+\mathrm{e}^{-2kH})$$

$$-2H\left[k\tanh(kH)-\nu\right]\mathrm{e}^{-2kH} \tag{35}$$

For convenience, introducing

$$Q(k)=2\nu+(k+\nu)\mathrm{e}^{-2kH} \tag{36}$$

Combining equations (34) and (35), the equation (36) becomes into

$$G_{01}=\int_0^{\infty}\mathrm{e}^{-kH}\left[\frac{Q(k)}{P(k)}-\frac{Q(K)}{(k-K)P'(K)}\right]\mathrm{e}^{k(z+\zeta+H)}\mathrm{J}_0(kR)\mathrm{d}k+$$
$$\frac{Q(K)}{P'(K)}\mathrm{P.V.}\int_0^{\infty}\frac{\mathrm{e}^{k(z+\zeta)}\mathrm{J}_0(kR)}{k-K}\mathrm{d}k \tag{37}$$

The first term of the above expression has no singularity which could be evaluated by Gauss-Laguerre integral. The second term could be evaluated by the expression of Green function in infinite water depth. Then

$$G_{01}=\sum_{i=1}^{n}\frac{w_i}{H}\left\{\left[\frac{Q(k_i)}{P(k_i)}-\frac{Q(K)}{(k_i-K)P'(K)}\right]\mathrm{e}^{k_i(z+\zeta+H)}\mathrm{J}_0(k_iR)\right\}$$
$$+\frac{Q(K)}{P'(K)}G_{\mathrm{inf}} \tag{38}$$

where $G_{inf}=\mathrm{P.V.}\int_0^{\infty}\frac{\mathrm{e}^{k(z+\zeta)}\mathrm{J}_0(kR)}{k-K}\mathrm{d}k$ is the expression in infinite water depth.

2.3 Series Solution

The above methods of Subsection integral and Gauss-Laguerre integral are time-consuming. 1984 Newman [3] has shown when $R/H\geq 0.5$, the series solution could obtain approximate value by a few terms, generally the term number is integer of $6H/R$. For large *n*,

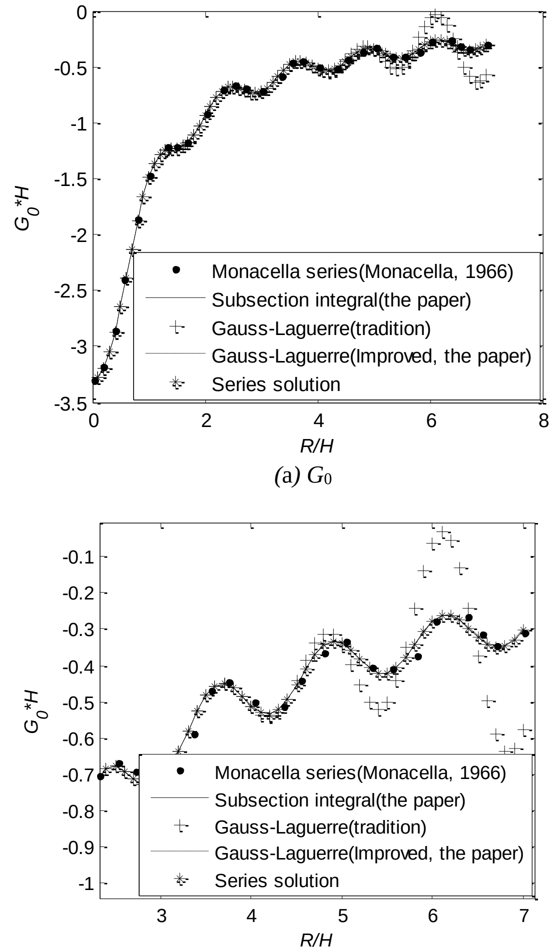
$$K_0(k_nR)=O\left(\mathrm{e}^{-\pi nR/H}\right) \tag{39}$$

The convergence rate of equation (2) depends on R/H. Especially, when $R/H=0$, the above expression would not converge.

It needs to obtain the plus root of equation $k_n\tan k_nH+\nu=0$, and k_n is between $(n-0.5)\pi/H$ and $n\pi/H$, n = 1, 2, 3… The roots could be obtained by dichotomy method.

3 NUMERICAL RESULTS

It is found that the value of singularity term of Green function has relationship with ω, *H*, *R*, ζ and *z* from the above derivations. After a certain non-dimensional derivatives of Green function G_0, it's concluded that the Green function only has relationship with $\bar{\nu}=\nu H$, $\bar{R}=R/H$, $\bar{z}=z/H$, $\bar{\zeta}=\zeta/H$. In order to verify the correctness of the method proposed in this paper, the value of Green function has been compared among different methods in the figures below. And some results from reference [8]. Additionally, the Gauss-Laguerre integral has used 30 points in the paper here. From the figures, the results of Subsection integral, Gauss-Laguerre (traditional) integral, Gauss-Laguerre (Improved) integral and series solution have good coincidence with results from reference [8]. Thus, the two methods propose by this paper have very high precision. Additionally, the traditional Gauss-Laguerre integral has high precision during *R/H* < 5.0, but loses precision during *R/H* > 5.0.



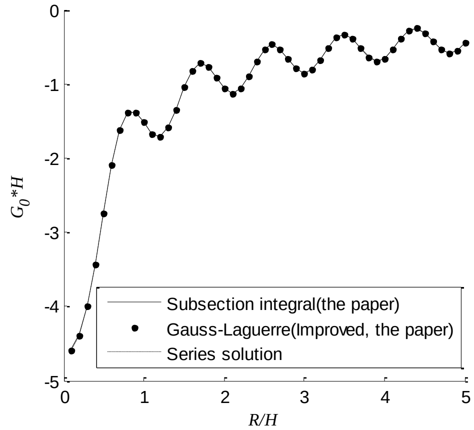
(a) G_0

(b) zoom out

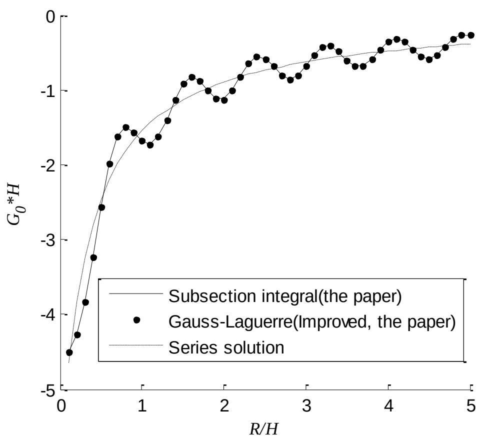
Figure 1. Green function G_0

($\bar{\nu}=\nu H$ =5.0, $\bar{z}=z/H=-1$, $\bar{\zeta}=\zeta/H=0$)

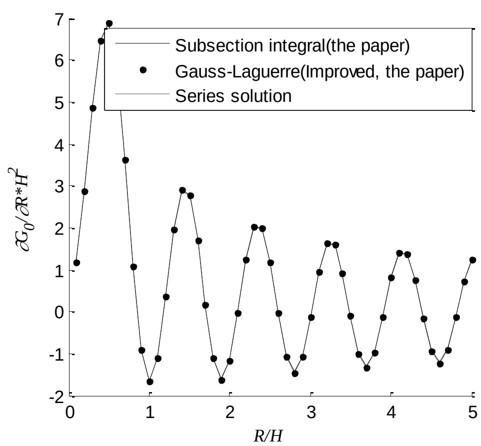
The principle of Green function G_0 and its derivatives varying by $\bar{\nu}$ is presented in the figures below, from which it's found that when $\bar{\nu}$ is very large, the series solution lose precision. And the results show that the unstable critical point is between 7.25 and 7.26.



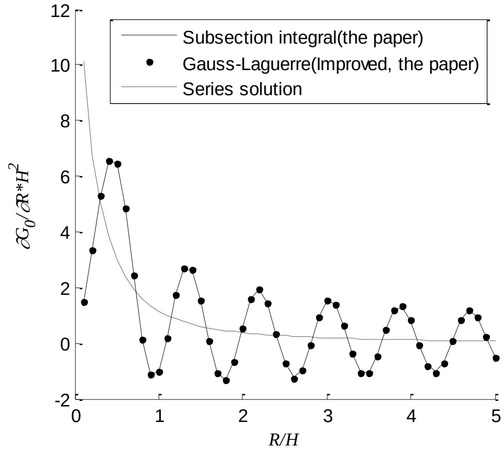
（a）G_0($\bar{\nu} = \nu H$ =7.0)



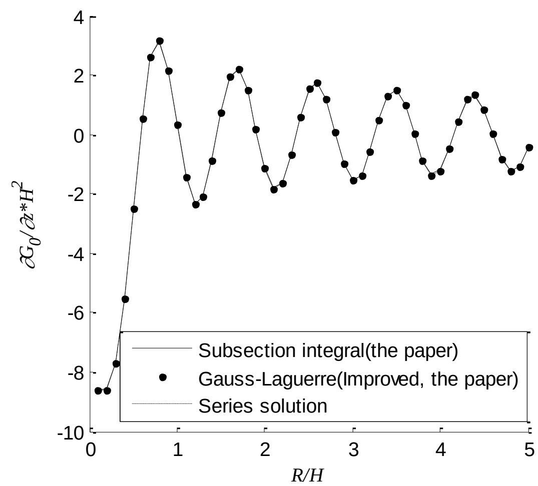
(b) G_0 ($\bar{\nu} = \nu H$ =7.5)



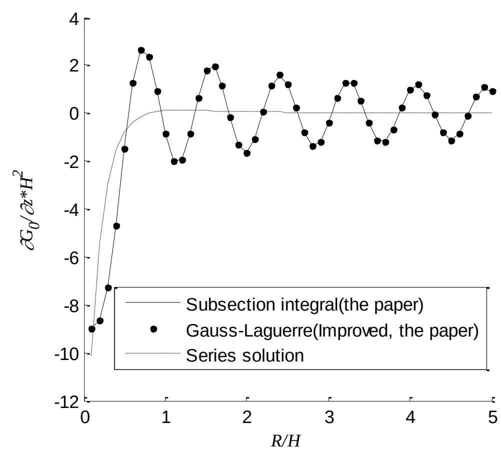
(c) dG_0 / dR ($\bar{\nu} = \nu H$ =7.0)



(d) dG_0 / dR ($\bar{\nu} = \nu H$ =7.5)



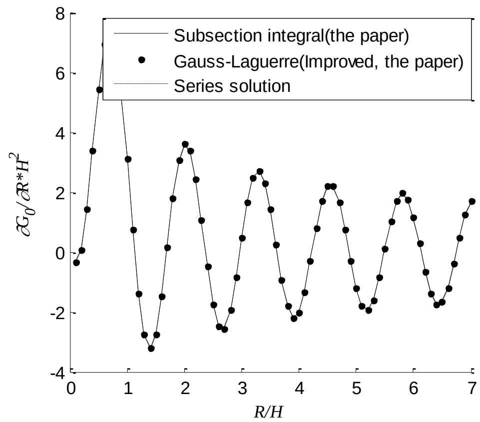
(e) dG_0 / dz ($\bar{\nu} = \nu H$ =7.0)



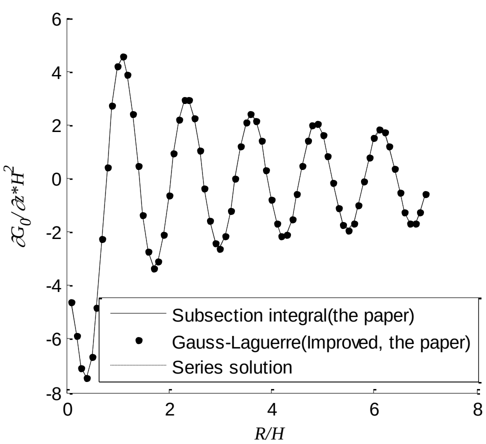
(f) dG_0 / dz ($\bar{\nu} = \nu H$ =7.5)

Figure 2. Green function G_0 ($\bar{z} = z / H = -0.5 , \bar{\zeta} = \zeta / H = 0$)

The derivatives dG_0 / dR and dG_0 / dx of Green function are presented in the figures below for the case $\bar{\nu} = \nu H$ =5.0. The results show that there are good coincidence among Subsection integral, Gauss-Laguerre(Improved) integral and series solution.



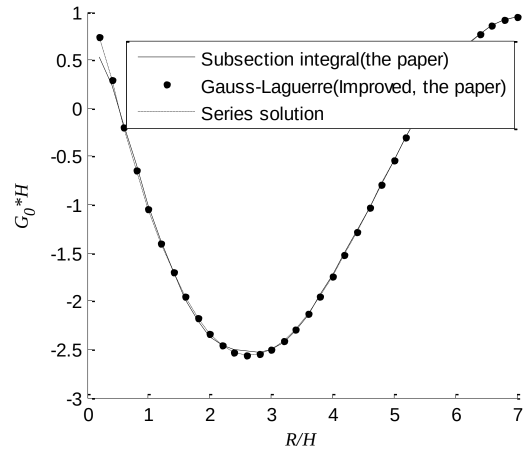
(a) dG_0 / dR



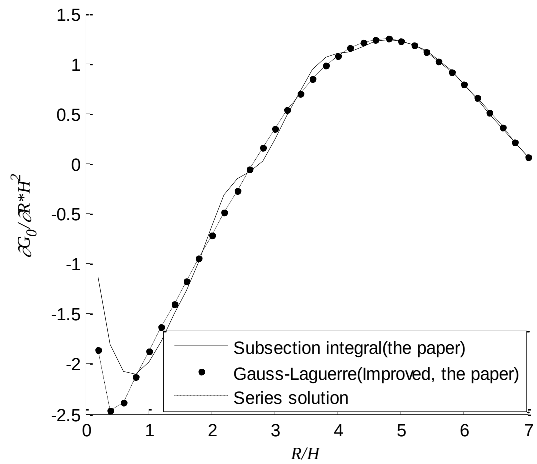
(b) dG_0 / dz

Figure 3. Derivatives of green function ($\bar{\nu} = \nu H$ =5.0, $\bar{z} = z / H = -0.5$, $\bar{\zeta} = \zeta / H = 0$)

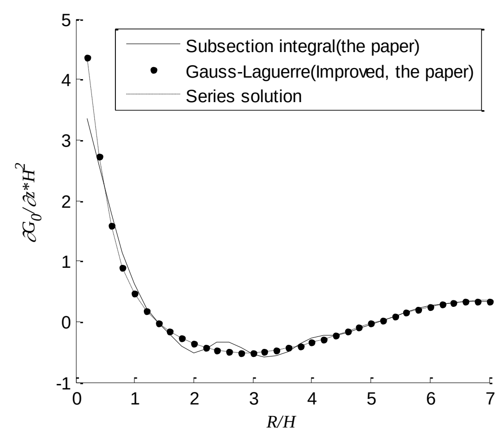
Although the results above show the two methods proposed in this paper and the series solution have high accuracy, these methods may lose precision in some special cases. It's concluded that the values of Green function and its derivatives by Subsection integral have big difference with Gauss-Laguerre(Improved) integral and series solution during R/H < 1.0 for $\bar{\nu}$ = 0.5 in figure 4, and at this time, Subsection integral method is not applicable. After a certain investigation, the reason is the oscillation of Bessel function, when R/H is small, the value of Bessel function decays very slow. It is impossible to evaluate the Green function by Subsection integral. On the other side, the Gauss-Laguerre(Improved) integral also may lose precision during *R*/*H* > 10.0 in figure 5. But this kind of error is no matter to obtain the accurate value of Green function for it could use series solution to evaluate the Green function and its derivatives when R/H > 0.5 which has included the case *R*/*H* > 10.0.



(a) G_0

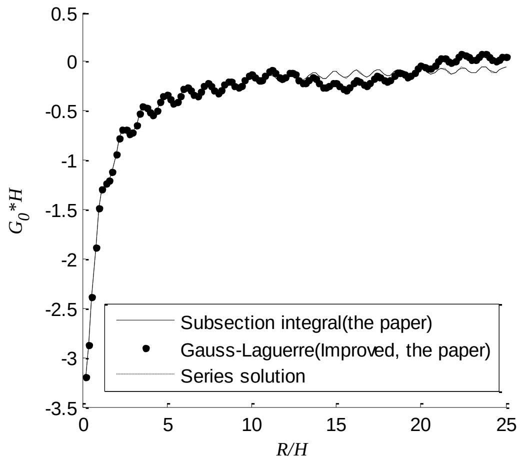


(b) dG_0 / dR

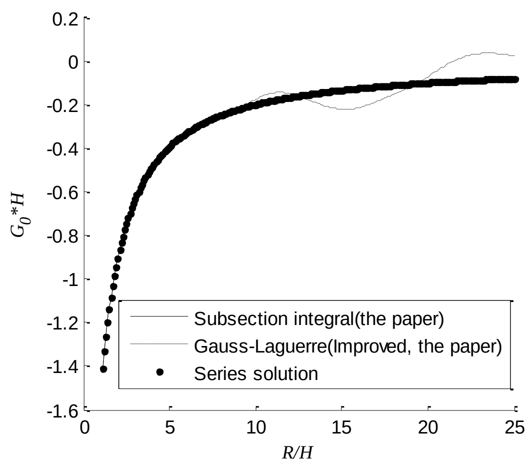


(c) dG_0 / dz

Figure 4. Green function and its derivatives ($\bar{\nu} = \nu H$ =0.5, $\bar{z} = z / H = -0.5$, $\bar{\zeta} = \zeta / H = 0$)



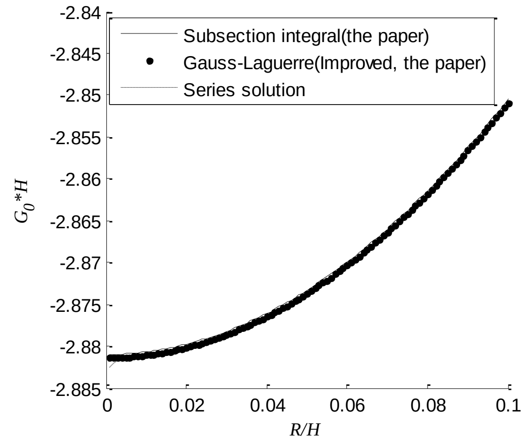
（a） $\bar{\nu} = \nu H$ =5.0



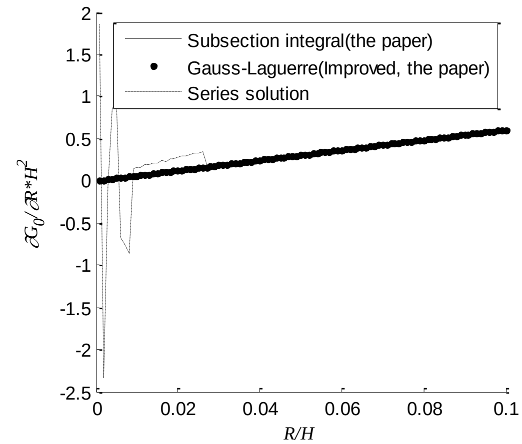
（b） $\bar{\nu} = \nu H$ =15.0

Figure 5. Green function G_0 ($\bar{z} = z/H = -1, \bar{\zeta} = \zeta/H = 0$)

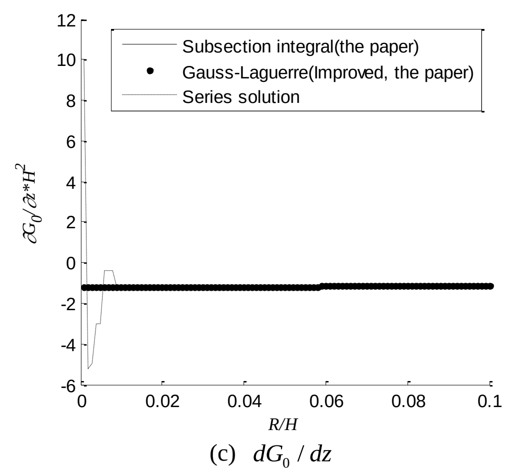
The following figures present the values of Green function and its derivatives during *R*/*H* approaching zero, which shows the two methods proposed in the paper has the same results, but series solution has difference with them. Additionally, it needs 300 terms in series solution to obtain proper result during *R*/*H* = 0.02, which is time-consuming.



(a) G_0



(b) dG_0 / dR



(c) dG_0 / dz

Figure 6. Green function and its derivatives
($\bar{\nu} = \nu H$ =7.0, $\bar{z} = z/H = -0.9$, $\bar{\zeta} = \zeta/H = 0$)

Furthermore, the above numerical results have shown that the series solution not only could obtain precise values of Green function and its derivatives during $R/H \ge 0.5$ which is just same as reference [3], but also could obtain the precise value during $0.1 \le R/H \le 0.5$. The series solution only needs term number of $[6H/R]$. When $0.1 \le R/H$, $[6H/R] \le 60$. The data from the below table have illustrated the efficiency is very high in the case. The total time-consuming of evaluating Green function and its derivatives are shown in table 1 by the methods of Subsection integral, improved Gauss-Laguerre integral and series solution, and the series solution has used 60 terms. The data in the table 1 show that the series solution is fastest, followed by improved Gauss-Laguerre integral, the Subsection integral is slowest. On the other side, the term number of series solution would increase by the decrease of R/H. When the term number is more than 300, the series solution would be slower than improved Gauss-Laguerre integral; meanwhile the former numerical results have shown that the series solution has lost precision in near-field of $R/H \le 0.02$.Thus it should adopt Gauss-Laguerre integral to calculate the value of Green function and its derivatives.

Table 1. Calculating velocity

Type	Subsection integral	Improved Gauss-Laguerre	Series solution (60terms)
Time-consuming(ms)	1.061	0.265	0.063

4 CONCLUDING REMARKS

The key of solving hydrodynamics of floating structures is to evaluate Green function and its derivatives. Generally, the integral form is adopted in near-field and the series solution is adopted in far-field. The calculating precision is compared among some methods, and which has verified that the two methods proposed in the paper could obtain the accuracy value of Green function and its derivatives in most cases. Then it's found that the series solution could not obtain precise value when νH is large. At this time it should adopt Subsection integral and improved Gauss-Laguerre integral. The series solution also would lose precision during *R/H* approaching zero. At this time it should adopt improved Gauss-Laguerre integral proposed by the paper. Although the two methods proposed in the paper could evaluate the value of Green function and its derivatives correctly, the method may lose precision in some special cases. For example, the method of Gauss-Laguerre integral may have a certain deviation when *R/H* is large. And the method of Subsection integral may lose precision when νH is large. Thus considering the precision and time-consuming in evaluating the Green function and its derivatives, one strategy has been given, that is the method of Gauss-Laguerre integral should be adopted when $R/H < 0.1$, and series solution is adopted when $R/H \ge 0.1$.

Acknowledgment

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5 REFERENCES

[1] Wehausen, J. V. and Laitone, E. V., 1960, "Surface Waves Handbuch der Physik", Berlin: *Springer-verleg.* pp. 446-778.

[2] John, F., 1950, "On the Motion of Floating Bodies II", *Comm. Pure Appl. Math.*, Vol. 3, pp. 45-101.

[3] Newman, J. N., 1985, "Algorithms for the Free-surface Green Function", *Journal of Engineering Mathematics*, Vol. 19, pp. 57-67.

[4] Li, L., 2001, "Numerical Seakeeping Predictions of Shallow Water Effect on Two Ship Interactions in Waves", PhD. Dissertation. *Dalhousie University*.

[5] Xie, Y. H., Li, R. P., Shu, Z., 2005, "Numerical Calculation of Finite Water-depth Composite Green Function", *Journal of Ship Mechanics*, Vol. 9(1), pp:23-28.

[6] Liu, R. M., Ren, H. L., Li, H., 2008, "An Improved Gauss-Laguerre Method for Finite Water Depth Green Function and Its derivatives", *Journal of Ship Mechanics*, Vol. 12(2), pp. 188-196.

[7] Wu, Y. S., 1984, "Hydroelasticity of Floating Bodies", PhD. *Brunel University,* pp. 184-185.

[8] Monacella, V. J., 1966, "The Disturbance Due to a Slender Ship Oscillating in Waves in a Fluid of Finite Depth", *Journal of ship research*, Vol. 10(4), pp. 242-252.